Gravitational Assist via Near-Sun Chaotic Trajectories of Binary Objects*

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Abstract

Using a process similar to the creation of hypervelocity stars, we show how binary objects (binary asteroid, spaceship+asteroid, etc.) can be used for interstellar travel. Previous research has shown that binary star - single star interactions can cause high-velocity ejection of one member of the inbound binary. By selecting the correct chaotic trajectory, the same should be attainable for ejecting the chosen member of a binary object targeted as near to the sun as is survivable by electronics and / or crew.

This paper will outline the basic process and compute the velocity that can be achieved under various orbital parameters via a conservation of energy calculation. We show via analogy to previously published calculations involving binary star - black hole interactions that suitable trajectories should exist to achieve useful energy gain.

1 Introduction

Gravitational assist is well known in spaceflight [10]. Simple gravitational assist from close approaches to moving bodies, typically planets, is used extensively for current unmanned space probes. These trajectories usually include an Oberth maneuver [12] to increase the acceleration during gravitational assist by firing engines at closest approach. In this paper, we discuss the possibility of another kind of gravitational assist achieved when two objects in mutual orbit make a close approach to a massive body.

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The interaction between binary stars and single objects has been been shown to allow for the ejection of one of the incoming objects at high velocity [9, 8]. The high stellar density of globular star clusters causes binary star - single star interactions to occur at a comparatively high rate. This interaction has been shown to cause the diameter of the core of globular star clusters to oscillate [3], sometimes chaotically [2], because of the ejection of stars as a heating source for the cluster.

Similarly, hypervelocity stars have been detected in the Milky Way galaxy [5]. Sixteen such stars are known to date. These are stars traveling at galactic escape velocities. The best explanation for these extreme velocities appears to be the interaction of a stellar binary with the Milky Way's central black hole [7].

This process is also suggested for how Triton was captured by Neptune [1]. And inbound binary object was tidally disrupted, capturing one element of the binary and ejecting the other. The same process has been proposed as a general mechanism for how planets can capture moons [14].

The question addressed here is whether binary objects can be used in a similar way for astronautics. By appropriately targeting a binary object toward a massive body, such as the Sun, we hope to eject one of those objects with a significant gain in velocity while leaving the other object in a close captured orbit. This paper computes the energy that can be transfered to one of the objects in the binary via a close approach to the Sun, and the resulting velocity obtained. This process leverages the intrinsically chaotic nature of a three-body system, so we expect desirable trajectories to be rare, if present. Leveraging the study by Hills [7] will show that suitable trajectories may, in fact, exist.

2 The Basic Trajectory

The key to the binary object gravitational assist maneuver is having a disposable object of significant mass. For purposes of discussion, we will assume that this is an asteroid in near-Earth-orbit (NEA). As of September 2011, 8,121 NEAs are known [11] with 828 of those having diameters of 1 km or greater. A recent discovery has shown that such objects exist in "Trojan" orbits [13] near the Earth, inhabiting the Earth-Sun Lagrangian points L4 and L5, just as happens with the gas giant planets. The first such object was found at roughly $+60°$ relative to the Earth sharing roughly the same orbit [6].

Using such an NEO, imagine that some part of the object is hollowed out and used as the interstellar space ship as proposed by D. Cole in the 1960s while the remainder is to be discarded. The NEO's orbit would have to be modified to achieve a close encounter with the sun.

En route to the Sun, the object would be split into a binary, presumably by destroying a "waist" that was created earlier in the object. Upon close approach to the Sun, assuming just the right trajectory, the expendable part would be captured into solar orbit and the "spaceship" would be ejected at high velocity.

The following results would also follow from assuming that a man-made spaceship enters orbit about an appropriately sized object, such as an asteroid

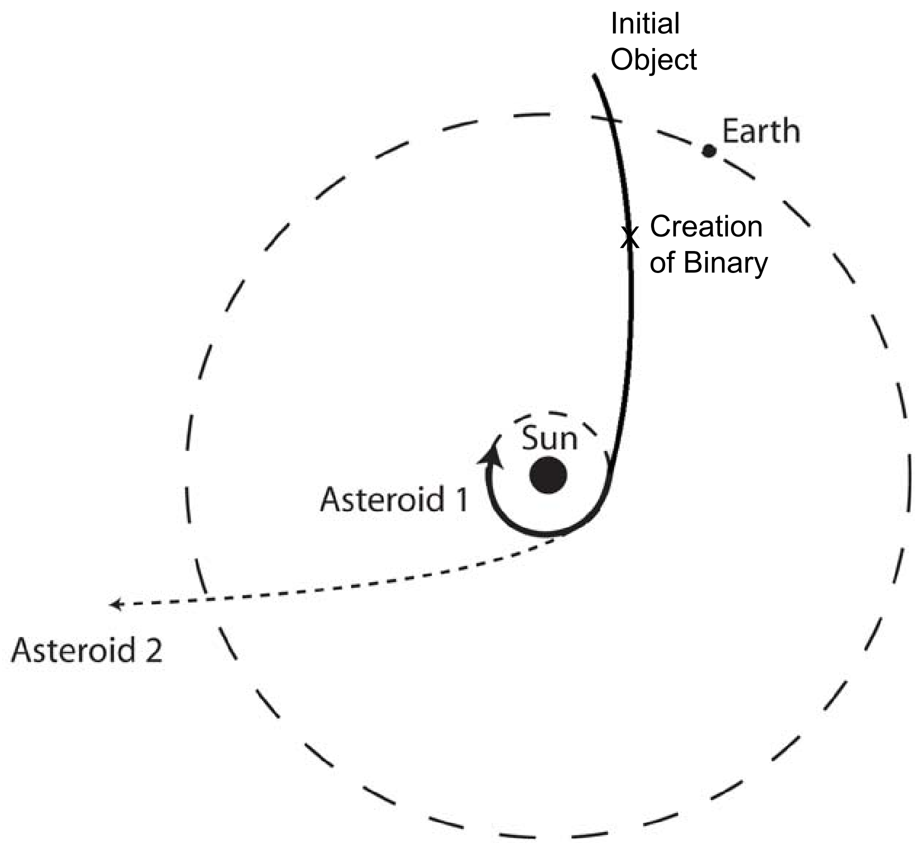

Figure 1: The assumed trajectory for the binary object. Starting from a near-Earth-orbit, a single object is assumed to be on a close approach trajectory to the Sun. En route, the object separates into Asteroid 1 that is assumed to be captured into a close orbit about the Sun, and Asteroid 2 that is assumed to be ejected at high velocity. Alternatively, Asteroid 1 could be the initial object and the second is a man-made spaceship placed in binary orbit with the asteroid.

or sun-grazing comet, and travels as a binary to near-Sun interaction.

3 Energy Transfer Calculations

This calculation looks at how much energy can be transfered from the binary to one of the final parts via a close approach to the Sun. The primary source of energy gain is the change in gravitational potential energy between near-Earth-orbit and perihelion. This is computed via a simple conservation of energy equation. Energy at aphelion equals energy at perihelion.

$$E_a = E_p \tag{1}$$

The initial energy will include orbital kinetic energy, rotational energy of the binary, and gravitational potential energy at aphelion.

$$\begin{aligned} E_a &= E_{a,orbital} + E_{rotational} + U_a && (2) \\ &= \frac{Gm_1m_2}{4R_b} + \frac{GMm_0}{2R_e} - \frac{GMm_0}{R_a} && (3) \end{aligned}$$

where

- m_1 is the mass of the object that will be captured into solar orbit.
- m_2 is the mass of the object that will be ejected.
- $m_0 = m_1 + m_2$ is the total mass of the initial binary.
- M is the mass of the Sun.
- R_b is the separation of the binary objects' mutual orbit.
- R_e is the mean radius of the Earth's orbit about the Sun.
- R_a is the distance of the object from the Sun at aphelion.

The equation for E_a would not normally include a kinetic energy term at aphelion if this is purely an object in an elliptical orbit. The generalization here allows for the use of residual kinetic energy from an orbital insertion maneuver such as a close flyby of a planet such as Earth.

When the binary object reaches closest approach to the Sun, if the right trajectory is attained, a transfer occurs that puts object 1 into close solar orbit and object 2 is ejected with total energy E_2.

$$\begin{aligned} E_p &= E_{1p,orbital} + U_{1p} + E_2 && (4) \\ &= \frac{GMm_1}{2R_p} - \frac{GMm_1}{R_p} + E_2 && (5) \end{aligned}$$

where

- R_p is the mean orbital radius of object 1 attained via the transfer at perihelion.
- E_2 is the total energy of object 2 after the transfer at perihelion.

Equating energy at aphelion with energy at perihelion after the transfer, we can solve for E_2 as

$$E_2 = G\left[\frac{m_1 m_2}{4R_b} + M\left(-\frac{m_0}{R_a} + \frac{m_0}{2R_e} + \frac{m1}{2R_f}\right)\right] \tag{6}$$

At infinity, the potential energy of object 2 will be zero, so the velocity of object 2 at infinity is

$$E_2 = \frac{1}{2} m_2 v_{2,\infty}^2 \tag{7}$$

giving

$$v_{2,\infty} = \left\{2G\left[\frac{m_1}{4R_b} + M\left(-\frac{1}{R_a}\frac{m_0}{m_2} + \frac{1}{2R_e}\frac{m_0}{m_2} + \frac{1}{2R_p}\frac{m_1}{m_2}\right)\right]\right\}^{\frac{1}{2}} \tag{8}$$

4 Results

Using Eq. 8 for the exit velocity of object 2, several things are immediately clear. The first term $\frac{m_1}{4R_b}$ comes from the rotational energy of the initial binary. That contribution is negligible compared to the other terms which are all multiplied by the Sun's mass.

Assuming that perihelion will be much closer than the Earth's orbit, only the final term will contribute significantly to the final result, so we find that

$$v_{2,\infty} \approx \sqrt{\frac{GM}{R_p}\frac{m_1}{m_2}} \tag{9}$$

This means that to maximize energy transfer, we want the smallest possible perihelion distance and for the "spaceship" mass m_2 to be much less than the mass of the deposited object m_1. Also note that the result is independent of the initial rotation of the binary and whether the original object's aphelion is greater than Earth's orbit, even out to 100 AU.

Using the full equation, we get the expected results in Figure 2.

With this information, we can approximate the travel time to Alpha Centari assuming no additional propulsion as shown in Figure 3.

When we consider the possibility of sending unmanned probes to another star, more extreme trajectories are possible. Current estimates are that electronics would survive a closest approach of 0.2 AU. If we could find a usable trajectory with perihelion of 0.2 AU for a probe with 0.1 mass ratio, the exit velocity would be 1,800 km/s or 0.06%c. With no other propulsion mechanism, that would correspond to a travel time of 7,300 years to Alpha Centari. This

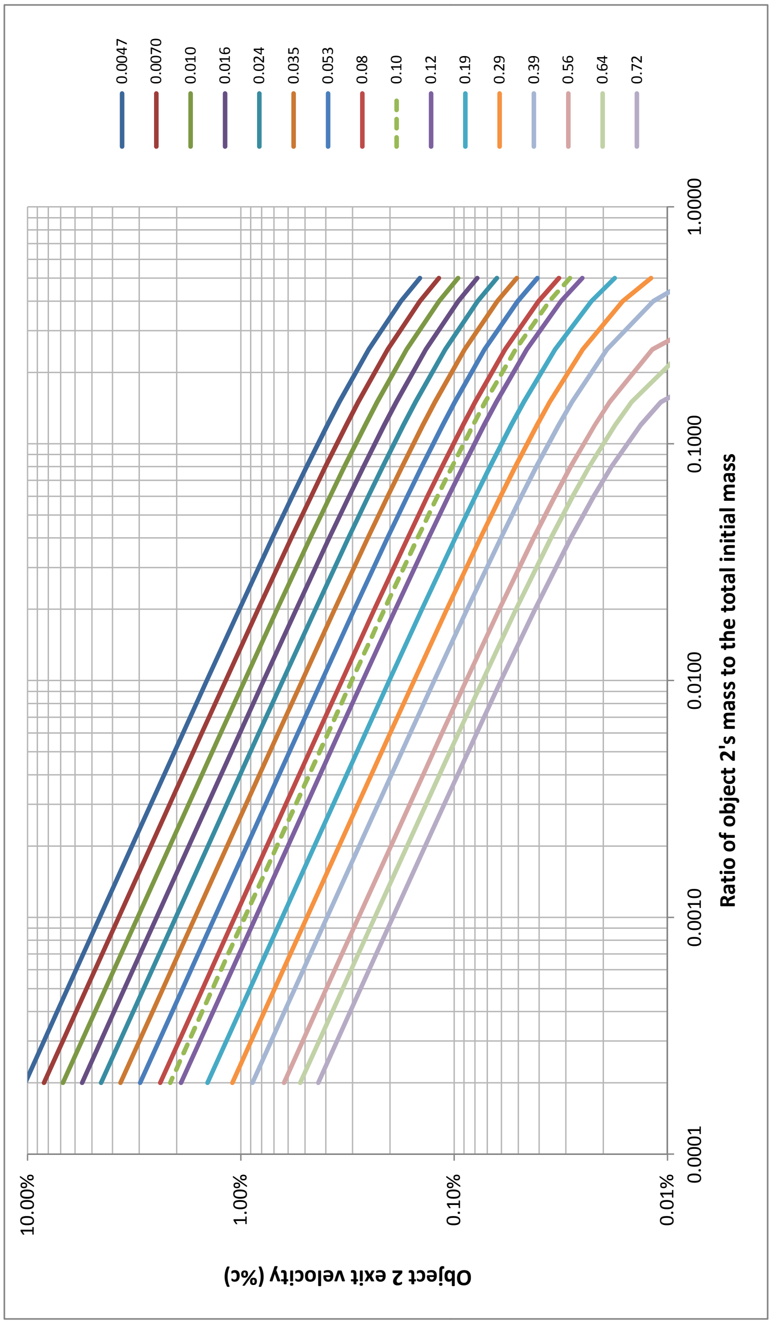

Figure 2: The exit velocity of object 2 at infinity as a fraction of the speed of light, c, for a range of masses, m_2. The lines correspond to perihelion distances in AU including $\frac{1}{4}$Mercury(0.10), Mercury(0.39), and Venus(0.72).

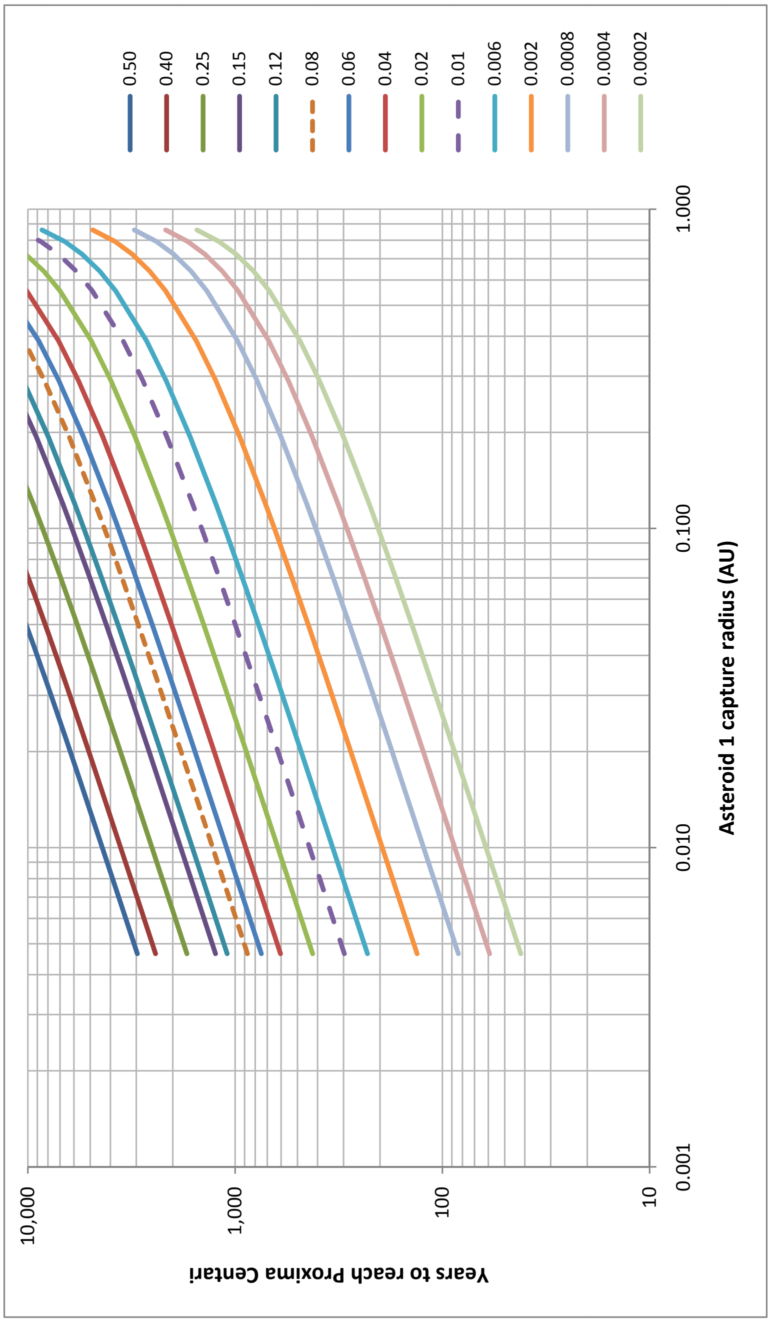

Figure 3: The travel time needed to reach Alpha Centari as a function of the perihelion distance. The lines correspond to the ratio of m_2/m_0.

result is highly sensitive to the perihelion distance and the mass ratio. Reducing the perihelion distance to 0.1 AU would increase the exit velocity by 50%. Conversely, increasing the mass ratio to 0.2 would cause an equivalent decrease in the exit velocity. Reducing the perihelion distance is largely an engineering question. The minimum mass ratio will be a physical constraint.

5 Probability of Ejection

The above calculation shows that sufficient energy is available. It does not prove that trajectories exist that can extract this energy. Hills [7] conducted high-fidelity simulations of binary star - black hole interactions and measured the probability of an ejection. Hills computes a dimensionless parameter, D_{min}, for which he shows the probability of an exchange. If we replace his black hole with our Sun and his binary star with our binary asteroid, we get following expression.

$$D_{min} = \frac{R_p}{R_b} \left(\frac{2M}{10^6 m_0} \right)^{-1/3} \tag{10}$$

According to Hills' simulations, for $D_{min} \approx 1$, the probability of interaction across all randomly selected initial conditions is 1%. This probability drops roughly linearly to 0 for $D_{min} \approx 170$. This implies that in order to have a non-zero probability of asteroid ejection, we need

$$\frac{R_p}{R_b} < 0.5 \tag{11}$$

In the current example, if we assume a total mass for the binary of $10^{15} kg$, which is approximately the mass of Deimos [4], a perihelion distance 0.2 AU, and a semi-major axis for the binary of 150 km, then the probability of an interaction is approximately 0.4%. This means that a detailed search will be required, but that ejection trajectories are possible.

Hills' formula does not consider the mass ratio of the objects in the initial binary. We must assume that for extreme mass ratios, ejection would not be observed. This is unfortunate since extreme ratios lead to the greatest ejection velocity. Specific simulations will be needed to determine the range of mass ratios over which ejection can occur.

The previous calculation of probability is based upon the assumption that Hills' expression can be translated to the current problem. That assumption should be confirmed via the same kind of simulation Hills conducted but made specific to our problem.

Although in theory one could achieve high exit velocities with real trajectories, we have not yet computed the G-forces that would be encountered by the ship and travelers. It may be possible that the acceleration encountered as these velocities are achieved are too great for the survival of ship or crew.

Also, we do not know how much margin for error exists in the computed trajectory. Even if the trajectories exist, are they navigable? At this point, we must assume that the mutual orbit of the binary must be precisely timed to its close approach of the Sun in order to get maximum energy transfer and be propelled in the correct direction. One must assume that some form of propulsion will be needed for course correction, and that precisions adjustments will be needed.

All of these questions could be resolved via simulations of specific orbits.

6 Conclusions

The calculations performed here show that a binary object-Sun gravitational assist maneuver could provide significant energy under plausible conditions. This could occur without the use of any significant propulsion in either engines or fuel. However, the most likely approach would incorporate elements of all known effects:

- Gravitational assist via a binary object-Sun interaction.
- Gravitational assist leveraging the proper motion of the Sun toward a nearby star like Alpha Centari.
- An Oberth maneuver at perihelion following ejection.

Just as important as getting to another star is being able to stop once you get there. One option would be to perform this binary object-star gravitational assist again, but in reverse, as was hypothesized for capture of moons by planets [1]. If the "spaceship" was a section of the original asteroid, one could envision splitting the remaining part again into a binary object and performing a close approach to the other star, but this time the crew would be in the portion captured into stellar orbit while ejecting a portion of their former home.

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